# ΑΠΑΝΤΗΣΕΙΣ

## ΘΕΜΑ Α

Α1) γ    Α2) δ    Α3) γ    Α4) β

Α5) Λ, Σ, Λ, Σ, Σ

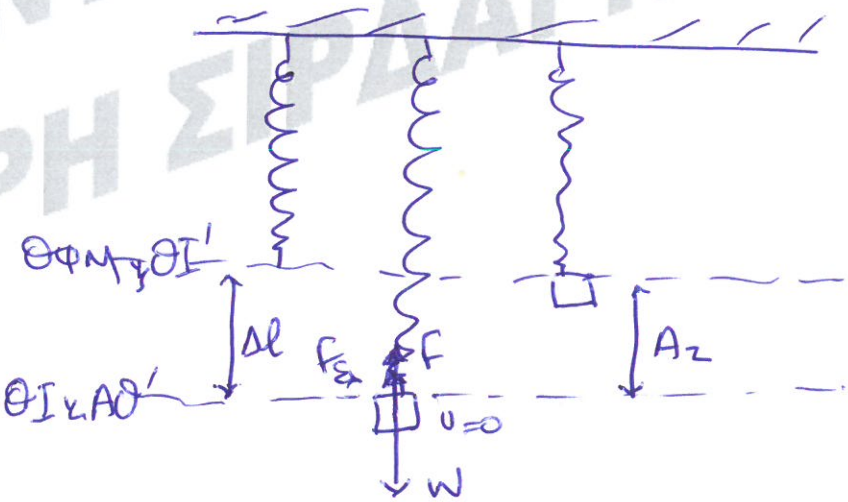
## ΘΕΜΑ Β

Β1) Αρχικά: ΘΦΜ ≡ ΑΘ  Άρα $\Delta\ell = A_1$

ΘΙ: $\Sigma F = 0 \Rightarrow w - F_{ελ} = 0 \Leftrightarrow mg = k\Delta\ell \Rightarrow \Delta\ell = \frac{mg}{k}$

$\Rightarrow \boxed{A_1 = \frac{mg}{k}}$

Τελικά:




ΘΙ': $\Sigma F = 0 \Leftrightarrow$

$F + F_{ελ} - w = 0 \Leftrightarrow$

$F_{ελ} = 0$ άρα

η ΘΙ' ≡ ΘΦΜ

ενώ η ΘΙ ≡ ΑΘ' άρα $A_2 = \Delta\ell = A_1 \Leftrightarrow \boxed{A_2 = A_1}$

Σωστό το (i)

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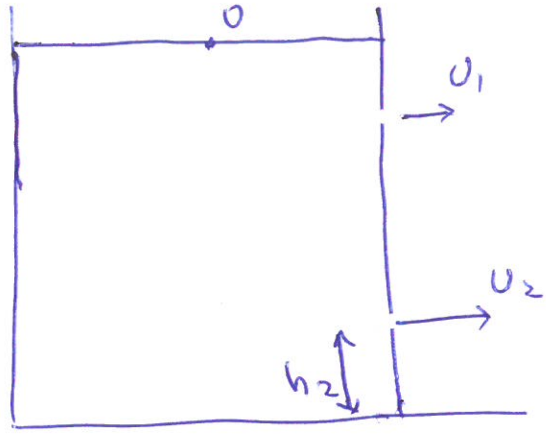
B2) $A_1 = A_2 = A$

$h_1 = \frac{5H}{6}$

$h_2 = \frac{H}{3}$




Bernoulli $0 \to 1$

$$υ_1 = \sqrt{2g(H-h_1)} \Rightarrow υ_1 = \sqrt{2g\frac{H}{6}} \Leftrightarrow υ_1 = \sqrt{\frac{gH}{3}} \quad (1)$$

Bernoulli $0 \to 2$

$$υ_2 = \sqrt{2g(H-h_2)} = \sqrt{2g\left(H-\frac{H}{3}\right)} \Leftrightarrow υ_2 = \sqrt{\frac{4gH}{3}} \quad (2)$$

(1) ÷ (2) $\quad \frac{υ_1}{υ_2} = \frac{\sqrt{\frac{gH}{3}}}{\sqrt{\frac{4gH}{3}}} = \frac{1}{2} \Leftrightarrow \boxed{υ_2 = 2υ_1} \Rightarrow$

$$\Rightarrow A\,υ_2 = 2Aυ_1 \Rightarrow Π_2 = 2Π_1 \quad (3)$$

Αρχικά $\quad Π_{ολ} = Π_1 = \frac{V}{Δt_1} \Rightarrow Δt_1 = \frac{V}{Π_1} \quad (4)$

Τελικά $\quad Π'_{ολ} = Π_1 + Π_2 \overset{(3)}{\Leftrightarrow} Π'_{ολ} = 3Π_1 = \frac{V}{Δt_2} \Rightarrow$

$\Rightarrow Δt_2 = \frac{V}{3Π_1} \quad (5) \qquad \frac{(4)}{(5)} \quad \frac{Δt_2}{Δt_1} = \frac{\frac{V}{3Π_1}}{\frac{V}{Π_1}} \Leftrightarrow \boxed{\frac{Δt_2}{Δt_2} = \frac{1}{3}}$

Σωστό το (ii)



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B3)

$$\left.\begin{aligned} P_1 &= m v_1 \\ P_1' &= \frac{P_1}{5} = \frac{m v_1}{5} \end{aligned}\right\} \text{Άρα } \quad v_1' = \frac{v_1}{5} \Rightarrow$$

$$\Rightarrow \frac{m_1 - m_2}{m_1 + m_2} \cancel{v_1} = \frac{\cancel{v_1}}{5} \Rightarrow 5m_1 - 5m_2 = m_1 + m_2 \Rightarrow$$

$$4m_1 = 6m_2 \Rightarrow m_1 = \frac{3}{2} m_2 \quad (2)$$

$$v_2' = \frac{2m_1}{m_1+m_2} v_1 = \frac{2\,\frac{3}{2}m_2}{\frac{3}{2}m_2+m_2} v_1 = \frac{3m_2}{5\frac{m_2}{2}} = \frac{6}{5}v_1 \Rightarrow$$

$$\Rightarrow v_2' = \frac{6}{5} v_1$$

$$\Pi = \frac{\Delta K_2}{K_1} \cdot 100\% = \frac{K_2' - \overset{0}{\cancel{K_2}}}{K_1} \cdot 100\% \Rightarrow$$

$$\Pi = \frac{\cancel{\frac{1}{2}} m_2 v_2'^2}{\cancel{\frac{1}{2}} m_1 v_1^2} \cdot 100\% = \frac{\cancel{m_2} \cdot \frac{36}{25} \cdot \cancel{v_1^2}}{\frac{3}{2}\cancel{m_2} \cdot \cancel{v_1^2}} \cdot 100\% = \frac{2 \cdot 36}{3 \cdot 25} \cdot 100\%$$

$$\boxed{\Pi = 96\%}$$

Σωστό το (iii)



ΦΡΟΝΤΙΣΤΗΡΙΑ ΑΡΓΥΡΗ ΣΙΡΔΑΡΗ

# ΘΕΜΑ Γ

$\ell = 1\,m$

$E = 9\,V$

$r = 1\,\Omega$

$m = 0,3\,kg$

$R_{\kappa\lambda} = 2\,\Omega$

Γ1) $B = ;$

$R_1 = 3\,\Omega$

$V_\Sigma = 6V$

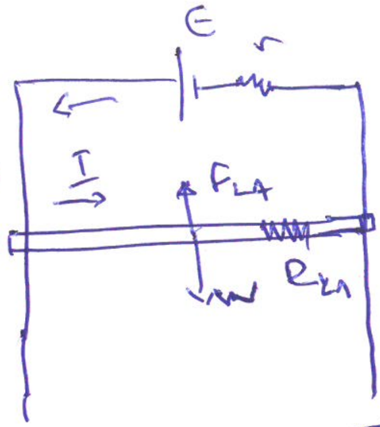
$P_\Sigma = 6W$

Γ2) $\upsilon_{o\rho} = ;$

Γ3) $\frac{dp}{dt} = ;$

$\upsilon = \frac{\upsilon_{o\rho}}{2}$

Γ1) Για να ισορροπεί ο αγωγός θα πρέπει η $\vec{F}_{LA}$ να έχει φορά προς τα πάνω. Άρα $\vec{B}$ ⊗




ΟΒ

$\Sigma F = 0 \Rightarrow F_{LA} = w \Leftrightarrow BI\ell = mg \Leftrightarrow$

$B \frac{E}{R_{o\lambda}} \ell = mg \Leftrightarrow B = \frac{mg(r + R_{\kappa\lambda})}{E \cdot \ell} \Leftrightarrow$

$\Rightarrow B = \frac{3 \cdot (1+2)}{9 \cdot 1} \Rightarrow \boxed{B = 1T}$

Γ2) $P_\Sigma = V_\Sigma I_\Sigma \Rightarrow \boxed{I_\Sigma = 1A}$

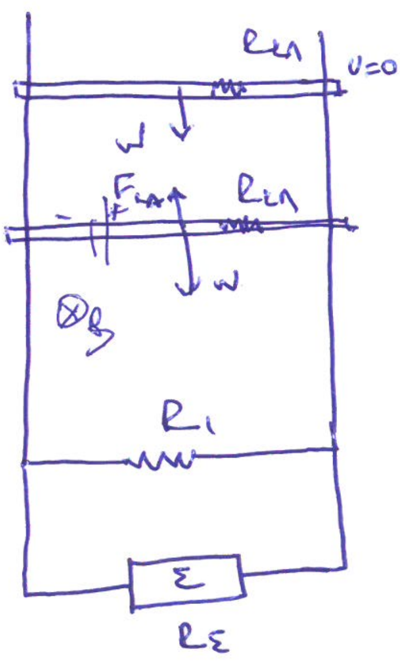
$I_\Sigma = \frac{V_\Sigma}{R_\Sigma} \Leftrightarrow \boxed{R_\Sigma = 6\,\Omega}$

Ισχύει

$R_{\iota\sigma} = \frac{R_1 \cdot R_\Sigma}{R_1 + R_\Sigma} \Rightarrow R_{\iota\sigma} = 2\,\Omega$

$R_{o\lambda} = R_{\iota\sigma} + R_{\kappa\lambda} \Leftrightarrow$

$\boxed{R_{o\lambda} = 4\,\Omega}$




Ισχύει γενικά $\Sigma F = ma \Rightarrow w - F_{LA} = ma$

$mg - BI_{\epsilon\pi}\ell = ma \Rightarrow$



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$$\Rightarrow mg - B\cdot\frac{E_{\varepsilon\pi}}{R_{o\lambda}}\ell = ma \Leftrightarrow mg - B\cdot\frac{B\upsilon\ell}{R_{o\lambda}}\ell = ma \Leftrightarrow$$

$$\Leftrightarrow 3 - \frac{\upsilon}{4} = 0{,}3a \Leftrightarrow a = 10 - \frac{\upsilon}{1{,}2} \Leftrightarrow a = 10 - \frac{5\upsilon}{6} \quad (SI) \quad (1)$$

Από τη σχέση (1) φαίνεται ότι όσο η ταχύτητα αυξάνεται η επιτάχυνση μειώνεται

Άρα το σώμα εκτελεί επιταχυνόμενη κίνηση με μειούμενη επιτάχυνση.

Όταν $\Sigma F = 0 \Rightarrow a = 0 \Rightarrow 10 - \frac{5\upsilon_{op}}{6} = 0 \Leftrightarrow$

$$\Leftrightarrow \boxed{\upsilon_{op} = 12\,m/s}$$

Γ3) $\upsilon = \frac{\upsilon_{op}}{2} \Leftrightarrow \upsilon = 6\,m/s$

(1) $a = 10 - \frac{5\cdot 6}{6} \Leftrightarrow a = 5\,m/s^2$

$$\frac{dp}{dt} = \Sigma F = ma = 0{,}3\cdot 5 \Rightarrow \boxed{\frac{dp}{dt} = 1{,}5\,kg\,m/s^2}$$

Γ4) Για $\upsilon_{op}$ ισχύει $I_{\varepsilon\pi} = \frac{E_{\varepsilon\pi}}{R_{o\lambda}} = \frac{B\upsilon_{op}\ell}{R_{o\lambda}} \Rightarrow I_{\varepsilon\pi} = 3A$

$V_\kappa = V_\Pi \Rightarrow V_\Pi = E_{\varepsilon\pi} - I_{\varepsilon\pi}R_{\varepsilon\sigma} \Leftrightarrow V_\Pi = B\upsilon_{op}\ell - I_{\varepsilon\pi}R_{\varepsilon\sigma}$

$\Rightarrow \boxed{V_\Pi = 6V}$ Όμως $V_\Pi = V_\Sigma = V_\kappa$ άρα λειτουργεί κανονικά



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# ΘΕΜΑ Δ

$M_\rho = 3\,kg$

$\ell = 2\,m$

$m = 1\,kg$

$\eta\mu\phi = 0,8$

$\sigma\upsilon\nu\phi = 0,6$

$M_T = 7\,kg$

$R = 0,4\,m$

$r = 0,3\,m$

Δ1) $T_1 = 10,5\,N$

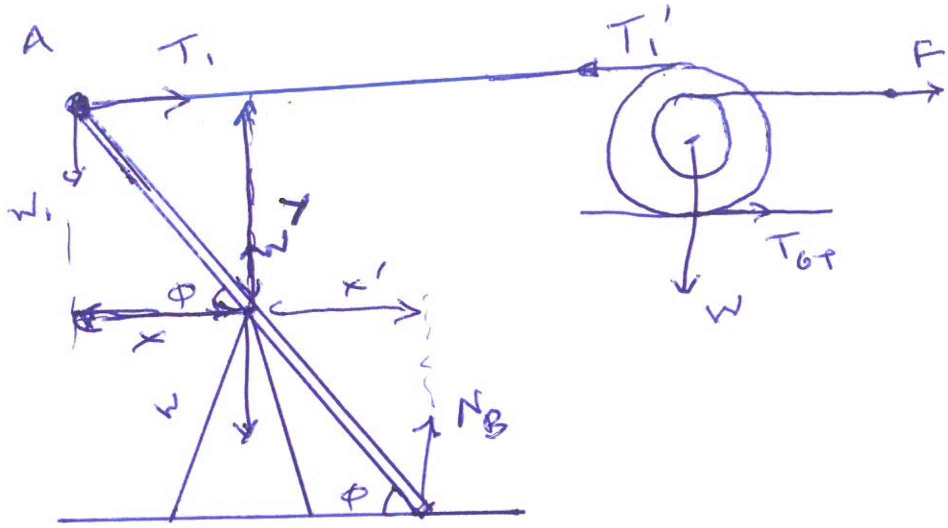
$N_B = ;$

Δ2) $\left(\frac{dL}{dt}\right)_{\mu\epsilon} = ;$

Δ3) $|\Delta L| = ;$

Δ4) $F = 12\,N$

$a_{cm} = ;$




Η ράβδος ισορροπεί

Δ1) $\Sigma\tau_{(\Gamma)} = 0 \Rightarrow \tau_{w_1} + \cancel{\tau_{N_\rho}}^{0} + \cancel{\tau_N}^{0} + \tau_{N_B} = 0$

$\Rightarrow w_1 \cdot x - T_1 y + N_B \cdot x' = 0 \qquad (1)$

$\eta\mu\phi = \frac{y}{\frac{\ell}{2}} \Leftrightarrow y = \frac{\ell}{2}\eta\mu\phi$

$\sigma\upsilon\nu\phi = \frac{x}{\frac{\ell}{2}} \Leftrightarrow x = \frac{\ell}{2}\sigma\upsilon\nu\phi$

$\sigma\upsilon\nu\phi = \frac{x'}{\frac{\ell}{2}} \Leftrightarrow x' = \frac{\ell}{2}\sigma\upsilon\nu\phi$

(1) $m_1 g \cdot \cancel{\frac{\ell}{2}}\sigma\upsilon\nu\phi - T_1 \cdot \cancel{\frac{\ell}{2}}\eta\mu\phi + N_B \cdot \cancel{\frac{\ell}{2}}\sigma\upsilon\nu\phi = 0$

$N_B \cdot 0,6 = 10,5 \cdot 0,8 - 10 \cdot 0,6 \Rightarrow$

$6N_B = 8 \cdot 10,5 - 6 \cdot 10 \Rightarrow \boxed{N_B = 4N}$

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Δ2) Για το σύστημα

$\Sigma\tau = I_{ολ}\cdot \alpha_{γων} \Leftrightarrow$

$\Rightarrow w_1 \cdot x = (I_ρ + I_1)\,\alpha_{γων} \Rightarrow$

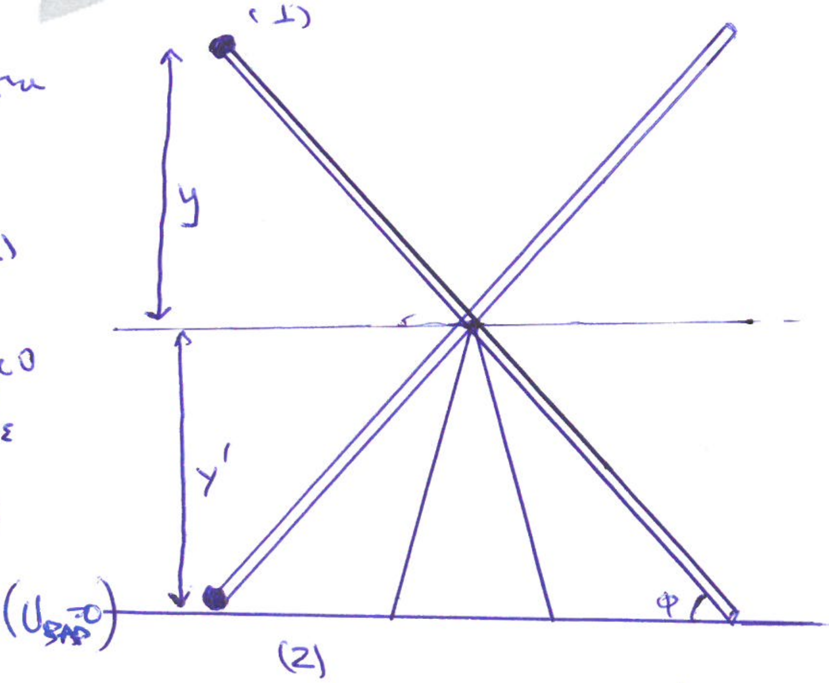
$\Rightarrow \alpha_{γων} = \dfrac{m_1 g \frac{\ell}{2}\cdot συνφ}{\frac{1}{12}M\ell^2 + m_1\left(\frac{\ell}{2}\right)^2} \Rightarrow$

$\Rightarrow \alpha_{γων} = \dfrac{m_1 g \frac{\ell}{2}\cdot συνφ}{\frac{1}{12}M\cancel{\ell^2} + m_1\frac{\cancel{\ell^2}}{4}} = \dfrac{1\cdot 10\cdot \frac{1}{2}\cdot 0,6}{\frac{1}{12}\cdot 3\cdot 2 + \frac{1}{2}} \Rightarrow \boxed{\alpha_{γων} = 3\,rad/s^2}$

Άρα για τη ράβδο $\left(\dfrac{dL}{dt}\right)_{ραβ} = I_ρ\cdot \alpha_{γων} = \dfrac{1}{12}ML^2\alpha_{γων}$

$\Rightarrow \left(\dfrac{dL}{dt}\right)_{ραβ} = \dfrac{1}{12}\cdot 3\cdot 2^2\cdot 3 \Rightarrow \boxed{\left(\dfrac{dL}{dt}\right)_{ραβ} = 3\,kg\,m^2/s^2}$

Δ3) Εφαρμόζω ΑΔΜΕ για το σύστημα από τη θέση (1) στη θέση (2)




$\cancel{K_1}^{\,0} + U_{1ρ} + U_{1Σ} = K_2 + U_{2ρ} + \cancel{U_{2Σ}}^{\,0}$

$\cancel{Mg}y' + mg(y'+y) = \frac{1}{2}I_{ολ}\omega^2 + \cancel{Mg}y'$

$mg(y'+y) = \frac{1}{2}\left(\frac{1}{12}M\ell^2 + m_1\frac{\ell^2}{4}\right)\omega^2 \Leftrightarrow$

$ημφ = \dfrac{y'}{\frac{\ell}{2}} \Rightarrow$

$y' = \frac{\ell}{2}ημφ$

$mg\left(\frac{\ell}{2}ημφ + \frac{\ell}{2}ημφ\right) = \frac{1}{2}\left(\frac{1}{12}M\ell^2 + m_1\frac{\ell^2}{4}\right)\omega^2$

ΦΡΟΝΤΙΣΤΗΡΙΑ ΑΡΓΥΡΗ ΣΙΡΔΑΡΗ

$\Rightarrow 10 \cdot 0{,}8 = \frac{1}{2}\left(\frac{1}{2}+\frac{2}{4}\right)\omega^2 \Rightarrow \boxed{\omega = 4\,rad/s}$

$L_{APX}$ ⊙

$L_{TE\Lambda}$ ⊗

⊗ (+)

$\Delta L = L' - L \Rightarrow \Delta L =$

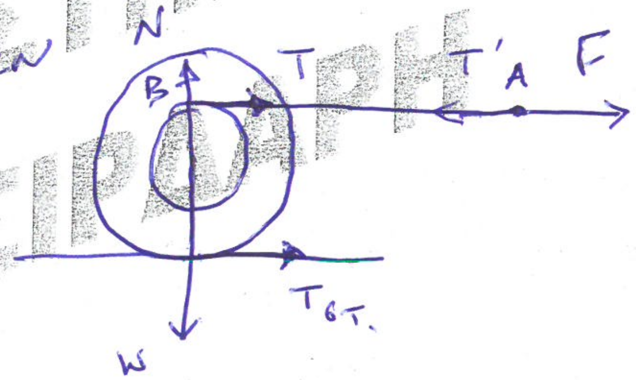
$\Delta L = I \cdot \omega' - (-I\omega) \Rightarrow$

$\Delta L = I \cdot \frac{\omega}{2} + I\omega \Rightarrow \Delta L = \frac{3}{2} \cdot I \cdot \omega \Rightarrow$

$\Delta L = \frac{3}{2} \cdot \left(\frac{1}{12}ML^2 + m\frac{\ell^2}{4}\right) \cdot \omega \Rightarrow \boxed{|\Delta L| = 12\,kg\,m^2/s}$

Με φορα $\Delta \vec{L}$ ⊗

Δ4) Ισχυει $\left.\begin{matrix} F = T' \\ T = T' \end{matrix}\right\} F = T = 12N$




$\Sigma F_x = M \cdot a_{cm} \Rightarrow$

$T + T_{στ} = M a_{cm}$ (2)

$\Sigma \tau = I a_{γων} \overset{εxo}{\Rightarrow} Tr - T_{στ}R = \frac{1}{2}MR^2 \cdot \frac{a_{cm}}{\cancel{R}}$ (εxo)

$\Rightarrow T \cdot \frac{3\cancel{R}}{4} - T_{στ}\cancel{R} = \frac{1}{2}M\cancel{R}a_{cm} \Rightarrow \frac{3T}{4} - T_{στ} = \frac{1}{2}Ma_{cm}$ (3)

(2) + (3) $\quad T + \frac{3T}{4} = \frac{1}{2}Ma_{cm} + Ma_{cm} \Rightarrow \frac{7T}{4} = \frac{3}{2}Ma_{cm} \Rightarrow$

$\Rightarrow \frac{\cancel{7} \cdot 12}{4} = \frac{3}{2} \cdot \cancel{7}a_{cm} \Rightarrow \boxed{a_{cm} = 2\,m/s^2}$

ΦΡΟΝΤΙΣΤΗΡΙΑ ΑΡΓΥΡΗ ΣΙΡΑΦΗ

Δ5) Ισχύει $υ_A = υ_B \Rightarrow υ_A = υ_{cm} + υ_{γρ} \Rightarrow$

$υ_A = ωR + ωr \Rightarrow υ_A = ωR + ω \cdot \frac{3R}{4} \Rightarrow$

$υ_A = \frac{7}{4} ωR \Leftrightarrow υ_A = \frac{7}{4} υ_{cm} \Leftrightarrow \frac{dυ_A}{dt} = \frac{7}{4} \frac{dυ_{cm}}{dt}$

$\Rightarrow a_A = \frac{7}{4} a_{cm}$ (4)

$W_F = F \cdot x_A = F \cdot \frac{1}{2} a_A \cdot t^2$

$W_F = F \cdot \frac{1}{2} \cdot \frac{7}{4} a_{cm} t^2 \Rightarrow W_F = 12 \cdot \frac{1}{2} \cdot \frac{7}{4} \cdot 2 \cdot 2^2$

$\boxed{W_F = 84 J}$

ΦΡΟΝΤΙΣΤΗΡΙΑ ΑΡΓΥΡΗ ΣΠΡΑΑΡΗ

Καλά Αποτελέσματα