ΑΠΑΝΤΗΣΕΙΣ

ΘΕΜΑ Α

Α1. γ

Α2. α

Α3. γ

Α4. δ

Α5. **α)** Σ, **β)** Λ, **γ)** Σ, **δ)** Σ, **ε)** Λ

ΘΕΜΑ Β

Β1. iii

Ο τροχός εκτελεί Κ.Χ.Ο. άρα:

$$\vec{\upsilon}_A = \vec{\upsilon}_{cm} + \vec{\upsilon}_{\gamma\rho} \Rightarrow |\upsilon_A| = \upsilon_{cm} + \upsilon_{\gamma\rho} \Rightarrow |\upsilon_A| = \omega R + \omega R \Rightarrow |\upsilon_A| = 2\omega R$$

$$\vec{\upsilon}_\Gamma = \vec{\upsilon}_{cm} + \vec{\upsilon}_{\gamma\rho} \Rightarrow |\upsilon_\Gamma| = \sqrt{\upsilon_{cm}^2 + \upsilon_{\gamma\rho}^2} \Rightarrow |\upsilon_\Gamma| = \sqrt{(\omega R)^2 + \left(\omega \frac{R}{2}\right)^2}$$

$$|\upsilon_\Gamma| = \sqrt{\omega^2 R^2 + \frac{\omega^2 R^2}{4}} \Rightarrow |\upsilon_\Gamma| = \sqrt{\frac{5}{4}\omega^2 R^2} \Rightarrow |\upsilon_\Gamma| = \frac{\omega R}{2}\sqrt{5}$$

$$\frac{|\upsilon_\Gamma|}{|\upsilon_A|} = \frac{\frac{\omega R}{2}\sqrt{5}}{2\omega R} \Rightarrow \frac{\upsilon_\Gamma}{\upsilon_A} = \frac{\sqrt{5}}{4}$$

Β2. ii

Πρώτη κρούση: $\upsilon_1' = \dfrac{m_1 - m_2}{m_1 + m_2}\upsilon_1$, $\upsilon_2' = \dfrac{2m_1\upsilon_1}{m_1 + m_2}$

$$\Pi_1\% = \frac{\Delta K_2}{K_{\alpha\rho\chi}} \cdot 100\% = \frac{\frac{1}{2}m_2\upsilon_2'^2}{\frac{1}{2}m_1\upsilon_1^2} \cdot 100\% = \frac{m_2 \frac{4m_1^2\upsilon_1^2}{(m_1+m_2)^2}}{m_1\upsilon_1^2} \cdot 100\%$$

$$\Pi_1\% = \frac{4m_1m_2}{\left(m_1+m_2\right)^2}\cdot 100\%$$

Δεύτερη κρούση: $\upsilon_2' = \frac{m_2-m_1}{m_1+m_2}\upsilon_2$, $\upsilon_1' = \frac{2m_2}{m_1+m_2}\cdot\upsilon_2$

$$\Pi_2\% = \frac{\Delta K_1}{K_{\alpha\rho\chi}}\cdot 100\% = \frac{\frac{1}{2}m_1\upsilon_1'^2}{\frac{1}{2}m_2\upsilon_2^2}\cdot 100\% = \frac{m_1\frac{4m_2^2}{\left(m_1+m_2\right)^2}\cdot\upsilon_2^2}{m_2\upsilon_2^2}\cdot 100\%$$

$$\Pi_2\% = \frac{4m_1m_2}{\left(m_1+m_2\right)^2}\cdot 100\%$$

$$\frac{\Pi_1}{\Pi_2} = \frac{\frac{4m_1m_2}{\left(m_1+m_2\right)^2}\cdot 100\%}{\frac{4m_1m_2}{\left(m_1+m_2\right)^2}\cdot 100\%} \Rightarrow \frac{\Pi_1}{\Pi_2} = 1 \Rightarrow \Pi_1 = \Pi_2$$

Β3. i

Toriccelli: $\upsilon_0 = \sqrt{2g\left(H-h_1\right)}$

$$h_1 = \frac{1}{2}gt_{o\lambda}^2 \Rightarrow t_{o\lambda} = \sqrt{\frac{2h_1}{g}}$$

$$s = \upsilon_0 t_{o\lambda} \Rightarrow s = \sqrt{2\cancel{g}\left(H-h_1\right)\frac{2h_1}{\cancel{g}}} \Rightarrow s = 2\sqrt{h_1\left(H-h_1\right)}$$

Οριζόντια βολή

$Ox: \ \upsilon_0 = \sigma\tau\alpha\theta.$	$Oy: \ \upsilon_y = gt$
$x = \upsilon_0 t \Rightarrow t = \frac{x}{\upsilon_0}$	$y = \frac{1}{2}gt^2 \Rightarrow y = \frac{1}{2}g\frac{x^2}{\upsilon_0^2}$

Για την ενδιάμεση θέση ισχύει

$$x = \frac{s}{2}, \quad y = h_1 - h_2$$

$$h_1-h_2=\frac{1}{2}g\frac{\left(\frac{s}{2}\right)^2}{\upsilon_0^2}\Rightarrow h_1-h_2=\frac{s^2g}{8\upsilon_0^2}\Rightarrow$$

$$h_1-h_2=\frac{4h_1\left(H-h_1\right)g}{8\upsilon_0^2}\Rightarrow h_1-h_2=\frac{4h_1\cancel{\left(H-h_1\right)}\cancel{g}}{8\cdot 2\cancel{g}\cancel{\left(H-h_1\right)}}\Rightarrow$$

$$h_1-h_2=\frac{h_1}{4}\Rightarrow\frac{3h_1}{4}=h_2\Rightarrow h_1=\frac{4}{3}h_2\Rightarrow h_1=\frac{4}{3}\cdot\frac{21H}{32}\Rightarrow$$

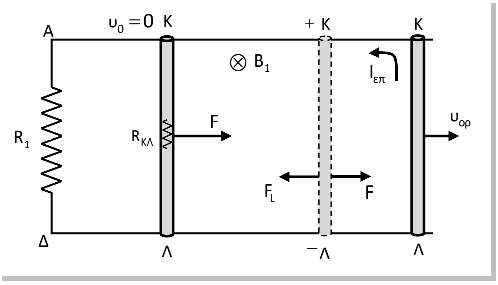
$$h_1=\frac{7H}{8}.$$

Άρα $\Pi=A\cdot\upsilon\Rightarrow\Pi=A\sqrt{2g\left(H-h_1\right)}\Rightarrow$

$$\Pi=A\sqrt{2g\left(H-\frac{7H}{8}\right)}\Rightarrow\Pi=A\sqrt{2g\frac{H}{8}}\Rightarrow\boxed{\Pi=\frac{A}{2}\sqrt{gH}}$$

ΘΕΜΑ Γ

Γ1.



$$E_{επ}=\frac{|d\Phi|}{dt}=\frac{Bds}{dt}=\frac{Bdx\cdot L}{dt}\Rightarrow E_{επ}=B\upsilon\ell$$

$$i_{επ}=\frac{E_{επ}}{R_{ολ}},\quad F_L=Bi_{επ}\ell,\quad R_{ολ}=R_1+R_{ΚΛ}$$

Την $t_0=0\rightarrow\upsilon_0=0$. Αρχίζει να ασκείται δύναμη F οπότε ο αγωγός αρχίζει να κινείται προς τα δεξιά και δημιουργείται $E_{επ}$ καθώς μεταβάλλεται ο εμβαδόν με την κίνησή του.

$\upsilon\uparrow,\ E_{επ}\uparrow,\ i_{επ}\uparrow,\ F_L\uparrow,\ |\Sigma F|=|F-F_L|\downarrow,\ |\alpha|\downarrow$

Άρα η κίνηση που εκτελεί ο αγωγός είναι ευθύγραμμη επιταχυνόμενη με συνεχώς μειούμενη κατά μέτρο επιτάχυνση.

Κάποια στιγμή $\Sigma F = 0 \Rightarrow F = F_L \Rightarrow \alpha = 0 \Rightarrow \upsilon = \upsilon_{o\rho}$.

$$\Sigma F = m\alpha \Rightarrow \alpha = \frac{F - F_L}{m} = \frac{F - BI\ell}{m} = \frac{F - B\frac{E_{\varepsilon\pi}}{R_{o\lambda}}\ell}{m} \Rightarrow \alpha = \frac{F - \frac{B^2\upsilon\ell^2}{R_{o\lambda}}}{m}$$

όταν $\upsilon = \upsilon_{o\rho} \Rightarrow \alpha = 0$.

$$F - \frac{B^2\upsilon_{o\rho}\ell^2}{R_{o\lambda}} = 0 \Rightarrow \upsilon_{o\rho} = \frac{F \cdot R_{o\lambda}}{B^2 \cdot \ell^2} \Rightarrow \upsilon_{o\rho} = \frac{\frac{8}{10}\cdot 5}{1\cdot 1} \Rightarrow \boxed{\upsilon_{o\rho} = 4\,m/s}\,.$$

Γ2. Από $t_1 \rightarrow t_2$ Ε.Ο.Κ. με $\upsilon = \upsilon_{o\rho}$.

Τη στιγμή t_2 ισχύει

$$E_{\varepsilon\pi} = B_3\upsilon_{o\rho}\ell \Rightarrow E_{\varepsilon\pi} = 4V$$

$$I = \frac{E_{\varepsilon\pi}}{R_{o\lambda}} \Rightarrow I = \frac{4}{5}A$$

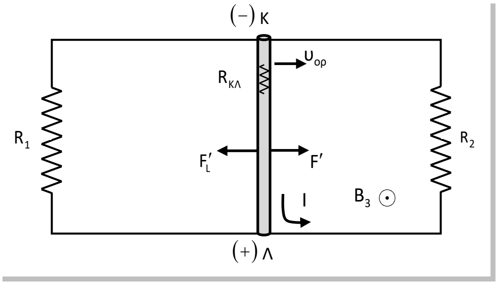
$$F_L = B_3 I\ell \Rightarrow F_L = 1\cdot\frac{4}{5}\cdot 1 \Rightarrow F_L = 0{,}8N$$

Πρέπει $\Sigma F = 0 \Rightarrow F' - F_L = 0 \Rightarrow \boxed{F' = 0{,}8N}$ με φορά προς τα δεξιά.

Γ3. $I = \frac{q}{\Delta t} \Rightarrow \Delta t = \frac{0{,}2}{0{,}8} \Rightarrow \Delta t = 0{,}25s$

$$Q = I^2 R_{o\lambda}\Delta t \Rightarrow Q = 0{,}8^2 \cdot 5 \cdot 0{,}25 \Rightarrow \boxed{Q = 0{,}8J}$$

Γ4.



$$R'_{\varepsilon\xi}=\frac{R_1R_2}{R_1+R_2}=\frac{2\cdot 2}{2+2}\Rightarrow R'_{\varepsilon\xi}=1\Omega$$

$$R'_{o\lambda}=R_{K\Lambda}+R'_{\varepsilon\xi}\Rightarrow R'_{o\lambda}=4\Omega$$

$$\Sigma\vec{F}=m\vec{\alpha}\Rightarrow F'-F'_L=m\alpha\Rightarrow \alpha=\frac{F'-\dfrac{B^2\upsilon'_{o\rho}\ell^2}{R'_{o\lambda}}}{B^2\ell^2}$$ πρέπει $\alpha=0$.

$$\upsilon'_{o\rho}=\frac{F'\cdot R'_{o\lambda}}{B^2\cdot\ell^2}=\frac{\frac{8}{10}\cdot 4}{1\cdot 1}\Rightarrow \boxed{\upsilon'_{o\rho}=3{,}2\,m/s}$$

$$V_{\Lambda K}=E'_{\varepsilon\pi}-I'_{o\lambda}\cdot R_{K\Lambda}\Rightarrow V_{\Lambda K}=B\upsilon'_{o\rho}\ell-\frac{B\upsilon'_{o\rho}\ell}{R_{o\lambda}}\cdot R_{K\Lambda}\Rightarrow$$

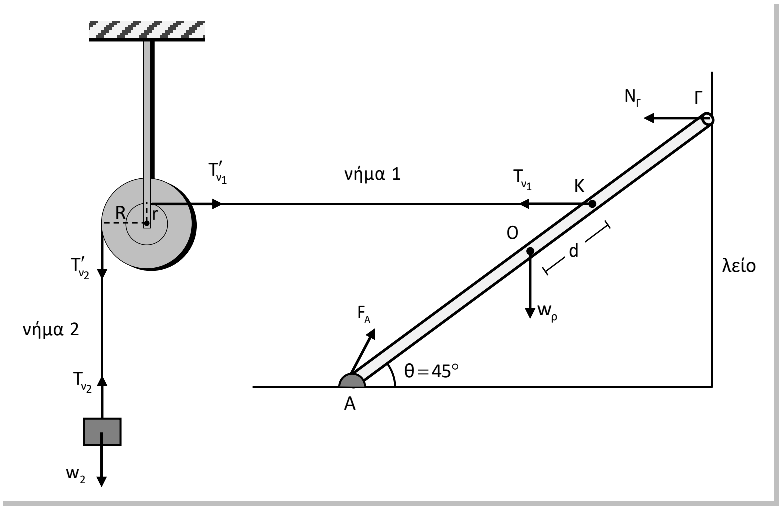
$$V_{\Lambda K}=3{,}2-0{,}8\cdot 3\Rightarrow V_{\Lambda K}=0{,}8V$$ άρα $\boxed{V_{K\Lambda}=-0{,}8V}$

$$I_1=\frac{V_{\Lambda K}}{R_1}=\frac{0{,}8}{2}\Rightarrow \boxed{I_1=0{,}4A}$$

$$I_2=\frac{V_{\Lambda K}}{R_2}\Rightarrow I_2=\frac{0{,}8}{2}\Rightarrow \boxed{I_2=0{,}4A}$$

ΘΕΜΑ Δ

Δ1.



$$\left|T_{\nu_1}\right| = \left|T'_{\nu_1}\right|, \quad \left|T_{\nu_2}\right| = \left|T'_{\nu_2}\right|$$

Ισορροπία m_2: $\Sigma F_2 = 0 \Rightarrow w_2 = T_{\nu_2} \Rightarrow T_{\nu_2} = 30N = T'_{\nu_2}$

Ισορροπία τροχαλίας: $\Sigma\tau = 0 \Rightarrow \tau_{T_{\nu_1}} = \tau_{T'_{\nu_2}} \Rightarrow T_{\nu_1} \cdot r = T'_{\nu_2} \cdot R \Rightarrow$

$$T_{\nu_1} \cdot r = T'_{\nu_2} \cdot 2r \Rightarrow T_{\nu_1} = 60N = T'_{\nu_1}$$

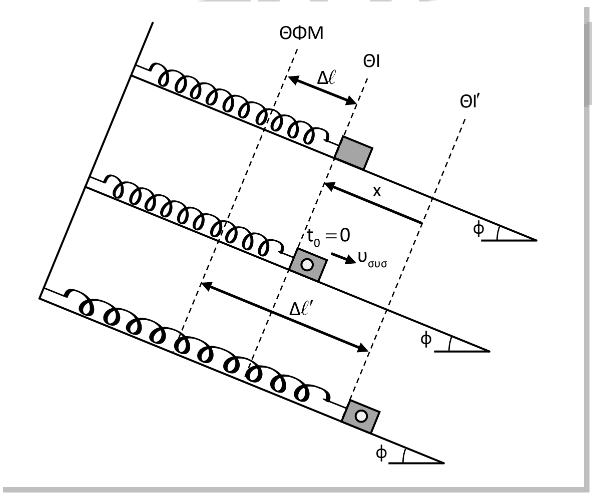
Ισορροπία ράβδου: $\Sigma\tau_A = 0 \Rightarrow$

$$-\tau_w + \tau_{N_\Gamma} + \tau_{T_{\nu_1}} = 0$$

$$-w\frac{\ell}{2}\cancel{\sigma\upsilon\nu\theta} + N_\Gamma \cdot \ell\,\cancel{\eta\mu\theta} + T_{\nu_1}\left(\frac{\ell}{2} + d\right)\cancel{\eta\mu\theta} = 0 \Rightarrow$$

$$N_\Gamma \cancel{\ell} + T_{\nu_1}\frac{4\cancel{\ell}}{6} = w_\rho\frac{\cancel{\ell}}{2} \Rightarrow N_\Gamma = -40 + 50 \Rightarrow \boxed{N_\Gamma = 10N}$$

Δ2.



Θ.Ι.: $\Sigma F = 0 \Rightarrow m_1 g\eta\mu\varphi = k\Delta\ell \Rightarrow \Delta\ell = 0{,}05m$

Θ.Ι.΄: $\Sigma F = 0 \Rightarrow (m_1 + m_2)g\eta\mu\varphi = k\Delta\ell' \Rightarrow \Delta\ell' = 0{,}2m$

$|x| = |\Delta\ell - \Delta\ell'| \Rightarrow |x| = 0{,}15m$.

<u>Α.Δ.Ε.Τ.</u>

$$E = K + U \Rightarrow \frac{1}{2}kA^2 = \frac{1}{2}(m_1 + m_2)\upsilon^2_{\sigma\upsilon\sigma.} + \frac{1}{2}kx^2 \Rightarrow$$

$$A^2 = \frac{\frac{36}{4}}{100} \Rightarrow \boxed{A = 0,3m}$$

Δ3. $k = (m_1 + m_2)\omega^2 \Rightarrow \omega = 5\, {}^{r}\!/_{s}$

Την $t = 0$, $x = -0,15m$, $\upsilon > 0$

$$-\frac{A}{2} = A\eta\mu\varphi_0 \Rightarrow \eta\mu\varphi_0 = \eta\mu\left(-\frac{\pi}{6}\right)$$

$$\varphi_0 = 2\kappa\pi - \frac{\pi}{6} \overset{\kappa=1}{\Rightarrow} \varphi_0 = \frac{11\pi}{6} \text{ δεκτή } \upsilon > 0$$

$$\varphi_0 = 2\kappa\pi + \pi + \frac{\pi}{6} \Rightarrow \varphi_0 = \frac{7\pi}{6} \text{ απορρίπτεται } \upsilon < 0$$

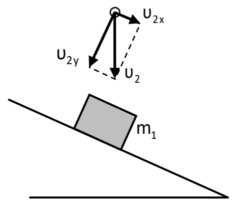
άρα $\boxed{x = 0,3\eta\mu\left(5t + \frac{11\pi}{6}\right) \ (S.I.)}$

Δ4. ΑΔΟ $(x'x)$

$m_2\upsilon_{2x} + 0 = (m_1 + m_2)\upsilon_{συσ.}$

$m_2\upsilon_2\eta\mu 30° = (m_1 + m_2)\upsilon_{συσ.} \Rightarrow$

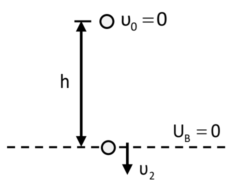
$$\upsilon_2 = \frac{4\frac{3\sqrt{3}}{4}}{3\cdot\frac{1}{2}} \Rightarrow \boxed{\upsilon_2 = 2\sqrt{3}\, {}^{m}\!/_{s}}$$



ΑΔΜΕ (m_2)

$$K_{αρχ}^{\,0} + U_{αρχ} = K_{τελ} + U_{τελ}^{\,0}$$

$$m_2gh = \frac{1}{2}m_2\upsilon_2^2 \Rightarrow h = \frac{4\cdot 3}{2\cdot 10} \Rightarrow \boxed{h = 0,6m}$$



Δ5. $$\frac{|F_{ελ_{max}}|}{|\Sigma F|} = \frac{|k(\Delta\ell' + A)|}{|-kA|} = \frac{0,5}{0,3} = \frac{5}{3} \Rightarrow \boxed{\frac{|F_{ελ_{max}}|}{|\Sigma F|} = \frac{5}{3}}$$