ΑΠΑΝΤΗΣΕΙΣ

ΘΕΜΑ Α

Α1. β, **Α2.** γ, **Α3.** β, **Α4.** δ
Α5. α – Σωστό, β – Λάθος, γ – Σωστό, δ – Λάθος, ε – Λάθος

ΘΕΜΑ Β
Β1. Σωστό το iii.

Άμεσα ακούει ο παρατηρητής: $f_1 = \frac{U_{HX}}{U_{HX}+U}\cdot f_S$.

Στο βράχο φτάνει: $f_{\beta\rho.} = \frac{U_{HX}}{U_{HX}-U}\cdot f_S$.

Και η ίδια $f_{\beta\rho}$ ανακλάται και φτάνει στον ακίνητο παρατηρητή. Άρα $f_2=f_{\beta\rho}$.

$$\frac{f_1}{f_2} = \frac{\frac{U_{HX}}{U_{HX}+U}\cdot f_S}{\frac{U_{HX}}{U_{HX}-U}\cdot f_S} = \frac{U_{HX}-U}{U_{HX}+U} = \frac{\frac{9}{10}\cdot U_{HX}}{\frac{11}{10}\cdot U_{HX}} = \frac{9}{11}$$

Β2. Σωστό το **i)**

$$A_M = 2A\cdot \sigma\upsilon\nu\frac{2\pi x_M}{\lambda} = 2A\cdot \sigma\upsilon\nu\frac{2\pi\frac{9\lambda}{8}}{\lambda} \Leftrightarrow$$

$$A_M = 2A\cdot \sigma\upsilon\nu\frac{2\pi 9}{8} = 2A\cdot \sigma\upsilon\nu\frac{9\pi}{4} = 2A\cdot \sigma\upsilon\nu\frac{\pi}{4}$$

$$U_M = \omega\cdot A_M = \frac{2\pi}{T}\cdot\left(2A\cdot \sigma\upsilon\nu\frac{\pi}{4}\right) = \frac{2\pi}{T}\cdot 2A\cdot\frac{\sqrt{2}}{2} \text{ άρα } U_M = \frac{2\cdot\sqrt{2}\cdot\pi A}{T}$$

Β3. Σωστό το **ii).**
Η παροχή είναι σταθερή: $\Pi_A = \Pi_B \Leftrightarrow A_A\cdot U_A = A_B\cdot U_B \Leftrightarrow 2\cdot \cancel{A_B}\cdot U_A = \cancel{A_B}\cdot U_B \Leftrightarrow$
$\Leftrightarrow \mathbf{U_B = 2\cdot U_A}$

Από την εξίσωση Bernoulli:

$$P_A + \frac{1}{2}\cdot\rho\cdot U_A^2 + \cancel{\rho\cdot g\cdot h_A} = P_B + \frac{1}{2}\cdot\rho\cdot U_B^2 + \cancel{\rho\cdot g\cdot h_B}$$

$$P_A - P_B = \frac{1}{2}\cdot\rho\cdot U_B^2 - \frac{1}{2}\cdot\rho\cdot U_A^2 \Leftrightarrow$$

$$P_A - P_B = \frac{1}{2}\cdot\rho\cdot(2U_A)^2 - \frac{1}{2}\cdot\rho\cdot U_A^2 \Leftrightarrow$$

$$P_A - P_B = 4\cdot\frac{1}{2}\cdot\rho\cdot U_A^2 - \frac{1}{2}\cdot\rho\cdot U_A^2 \Leftrightarrow P_A - P_B = 3\cdot\frac{1}{2}\cdot\rho\cdot U_A^2$$

ή PA – PB = **3 · Λ**

ΘΕΜΑ Γ

Γ1. Κάνουμε ΑΔΜΕ από το Α στο Γ.

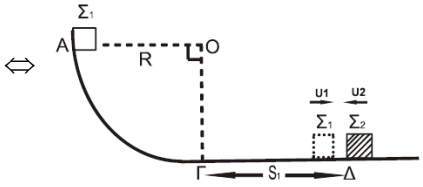
$$E_{M_{(\Gamma)}} = E_{M_{(A)}} \Leftrightarrow \cancel{U_{\beta_\Gamma}}^{0} + K_\Gamma = U_{\beta_A} + \cancel{K_A}^{0} \Leftrightarrow \frac{1}{2} m_1 U_\Gamma^2 = m_1 \cdot g \cdot R \Leftrightarrow U_\Gamma = \sqrt{2gR} \Leftrightarrow U_\Gamma = 10 m/s$$

Γ2. Κάνουμε ΘΜΚΕ από το Γ στο Δ (για m_1)

Wολ = ΔΚ ⇔ $W_T = K_{τελ} - K_{αρχ}$

Για m_1: ΣFy = 0 ⇔ $N_1 = W_1$ ⇔ $N_1 = m_1 \cdot g$

και $T_1 = \mu \cdot N_1 = \mu \cdot m_1 \cdot g$



Σχήμα 4

$$\Leftrightarrow -T_1 \cdot s = \frac{1}{2} \cdot m_1 \cdot U_1^2 - \frac{1}{2} \cdot m_1 \cdot U_\Gamma^2 \Leftrightarrow$$

$$-\mu \cdot \cancel{m_1} \cdot g \cdot s = \frac{1}{2} \cdot \cancel{m_1} \cdot U_1^2 - \frac{1}{2} \cdot \cancel{m_1} \cdot U_\Gamma^2 \Leftrightarrow U_1^2 = U_\Gamma^2 - 2\mu \cdot gs$$

$$U_1 = \sqrt{10^2 - 2 \cdot 0{,}5 \cdot 10 \cdot 3{,}6} = \sqrt{100 - 36} \Leftrightarrow U_1 = 8 m/s$$

Στην ελαστική κρούση: (από ΑΔΟ και ΑΔΕ)

$$U_1' = \frac{m_1 - m_2}{m_1 + m_2} \cdot U_1 + \frac{2m_2}{m_1 + m_2} \cdot U_2 = \frac{m_1 - 3m_1}{m_1 + 3m_1} \cdot U_1 + \frac{2 \cdot 3m_1}{m_1 + 3m_1} \cdot U_2$$

$$U_1' = \frac{-2m_1}{4m_1} \cdot 8 + \frac{6m_1}{4m_1} \cdot (-4) = -4 - 6 \Leftrightarrow U_1' = -10 m/s$$

$$U_2' = \frac{m_2 - m_1}{m_1 + m_2} \cdot U_2 + \frac{2m_1}{m_1 + m_2} \cdot U_1 = \frac{3m_1 - 3m_1}{m_1 + 3m_1} \cdot U_2 \Leftrightarrow U'_2 = -2 + 4 = 2 \frac{m}{s}$$

Άρα τα μέτρα των ταχυτήτων είναι: $|U_1'| = 10 m/s$

$|U_2'| = 2 m/s$

Γ3. $\Delta \vec{p}_2 = \vec{p}_2' - \vec{p}_2$

$$\Delta p_2 = (m_2 \cdot U_2') - (m_2 \cdot U_2) = (3 \cdot 2) - [3 \cdot (-4)]$$

Δp_2 = 6+12 = 18kg · m/s, **προς τα δεξιά**

και **$|\Delta p_2|$ =18kg · m/s,**

Γ4. $K_1 = \frac{1}{2} m_1 \cdot U_1^2 = \frac{1}{2} m_1 \cdot 8^2 = 32 \cdot m_1$ και $K'_1 = \frac{1}{2} m_1 \cdot U'^2_1 = \frac{1}{2} m_1 \cdot 10^2 = 50 \cdot m_1$

άρα $\Delta K_1\% = \frac{\Delta K_1}{K_1} \cdot 100\% = \frac{K'_1 - K_1}{K_1} \cdot 100\% =$

$$= \frac{50m_1 - 32m_1}{32 \cdot m_1} \cdot 100\% = \frac{18 \cdot m_1}{32 \cdot m_1} \cdot 100\% = \frac{9}{16} \cdot 100\% \Leftrightarrow \Delta K_1\% = 56{,}25\%$$

ΘΕΜΑ 4

m=1kg, k=100N/m, Φ= 30°, M= 2kg, R=0,1m, g = 10m/s², $Icm = \frac{1}{2} MR^2$, $ημ30 = \frac{1}{2}$

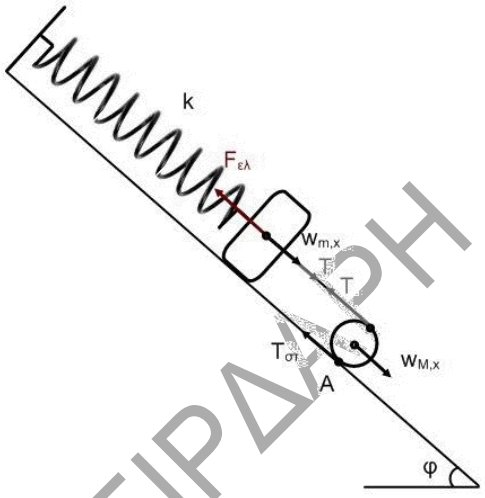
Δ1. Τ = ; , Δl = ; **Δ2.** ΣF(t) **Δ3.** L = ; για $N=\frac{12}{\pi}$περ **Δ4.** $\frac{dK}{dK}$; t= 3s

Δ1. **Για τον κύλινδρο**

$\Sigma\tau=0 \Rightarrow T_{\sigma\tau}R - TR = 0 \Rightarrow T = T_{\sigma\tau}$ (1)

$\Sigma F_x = 0 \Rightarrow Wx - T - T_{\sigma\tau} = 0 \Rightarrow Wx = T + T_{\sigma\tau} \Rightarrow$

$\overset{1}{\Leftrightarrow} W_x = 2T \Leftrightarrow Mg\eta\mu\varphi = 2T \Leftrightarrow$ **T = 5N**



Για το σώμα

$\Sigma F_x=0 \Leftrightarrow F_{\epsilon\lambda} - T - Wx = 0 \Leftrightarrow K\Delta l - T - Wx = 0$

$\Leftrightarrow K\Delta l = T + mg\eta\mu\varphi \Leftrightarrow \Delta l = \frac{5+1\cdot 10\frac{1}{2}}{100} \Leftrightarrow$

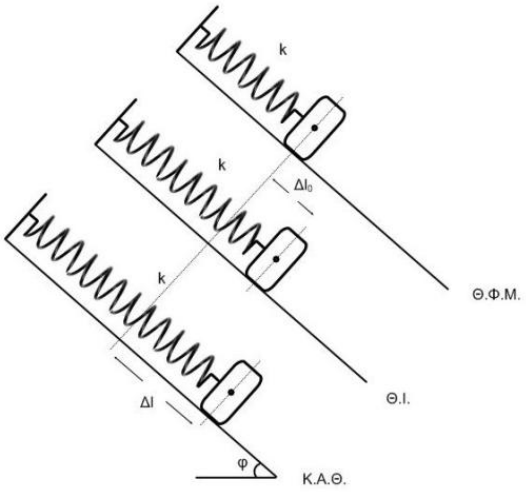
$\Leftrightarrow$ **Δl = 0,1m**

Δ2. Όταν κοπεί το νήμα η ΘΙ θα γίνει ΑΘ.

Για τη νέα ΘΙ΄ ισχύει $\Sigma F_x = 0 \Leftrightarrow Wx = F'_{\epsilon\lambda} \Leftrightarrow$

$\Leftrightarrow mg\eta\mu\varphi = K\Delta l' \Leftrightarrow \Delta l' = \frac{5}{100} \Leftrightarrow$ **Δl΄ = 0,05m**

Άρα A = Δl - Δl΄ $\Rightarrow$ **A = 0,05m**



Ισχύει x = Aημ(ωt+φ_o)
t = 0
x = - A
} $-A = A\eta\mu\varphi_o \Rightarrow \eta\mu\varphi=-1 \Rightarrow$

$\Rightarrow \eta\mu\varphi_o = \eta\mu\frac{3\pi}{2} \Leftrightarrow \varphi_o = \frac{3\pi}{2}$

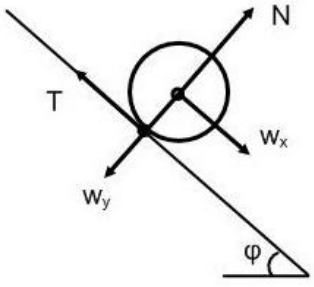
Επίσης $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{1}} \Leftrightarrow \omega = 10rad/s$

Άρα $x= 0,05\eta\mu\left(10t+\frac{3\pi}{2}\right)$ (S,I)

Για την δύναμη $\Sigma F = -Kx \Leftrightarrow \Sigma F = -100 \cdot 0,05\eta\mu\left(10t+\frac{3\pi}{2}\right)$

$\Leftrightarrow \Sigma F = -5\eta\mu\left(10t+\frac{3\pi}{2}\right)$ (S, I)

Δ3. **Για τον κύλινδρο**



Κ.Χ.Ο. άρα $x = S = \theta \cdot R$ (2)

$\upsilon_{cm} = \omega \cdot R$ (3)

$a_{cm}=a_{\gamma\omega\nu} \cdot R$ (4)

$\Sigma F = M\cdot\alpha_{cm} \Leftrightarrow Wx-T_{\sigma\tau} = M\cdot\alpha_{cm} \Leftrightarrow Mg\eta\mu\varphi-T_{\sigma\tau} = M\cdot\alpha_{cm}$

$\Leftrightarrow 10 - T_{\sigma\tau} = 2\alpha_{cm}$ (5)

$\Sigma\tau = I\cdot\alpha_{\gamma\omega\nu.} \overset{(4)}{\Leftrightarrow} T_{\sigma\tau}\cdot R = \frac{1}{2}MR^2\frac{\alpha_{cm}}{R} \Leftrightarrow T_{\sigma\tau} = \alpha_{cm}$ (6)

(5) $\overset{(6)}{\Leftrightarrow} 10-\alpha_{cm}=2\alpha_{cm} \Leftrightarrow \alpha_{cm} = \frac{10}{3}m/s^2$

$N = \frac{S}{2\pi R} \Leftrightarrow S = \frac{12}{\pi}2\pi\cdot 0,1 \Leftrightarrow S = 2,4m$

$S = \frac{1}{2}\alpha_{cm}t^2 \Leftrightarrow t = \sqrt{\frac{2S}{a_{cm}}} = \sqrt{\frac{2\cdot 2,4}{\frac{10}{3}}} \Leftrightarrow t = \sqrt{1,44} \Leftrightarrow t = 1,2s$

$U = \alpha_{cm}t = \frac{10}{3}\cdot 1,2 \Leftrightarrow U = 4m/s$

Άρα $L = I\omega = \frac{1}{2}MR^2\frac{U_{cm}}{R} = \frac{1}{2}\cdot 2\cdot 0,1\cdot 4 \Leftrightarrow$ **$\boxed{L=0,4kgm^2/s}$**

Δ4. Για t=3s και (6) $T_{\sigma\tau} = \frac{10}{3}N$

$U = \alpha_{cm}\cdot t = \frac{10}{3}\cdot 3 \Leftrightarrow U_{cm} = 10m/s$

$\frac{dK}{dt} = \frac{dW}{dt} = \frac{d\Sigma F\cdot x}{dt} + \frac{d\Sigma\tau\cdot\theta}{dt} = \Sigma F\cdot Ucm + \Sigma\tau\cdot\omega \Leftrightarrow$

$\Leftrightarrow \frac{dK}{dt} = (Wx - T_{\sigma\tau})Ucm + T_{\sigma\tau}\cdot R\frac{Ucm}{R} \Leftrightarrow$

$\Leftrightarrow \frac{dK}{dt} = \left(10-\frac{10}{3}\right)\cdot 10 + \frac{10}{3}\cdot 10 \Leftrightarrow \frac{dK}{dt} = \frac{200}{3} + \frac{100}{3} \Leftrightarrow$

$\Leftrightarrow \frac{dK}{dt} = 100\,J/s$